NCERT Solutions Class 8 Maths (Ganita Prakash)

Chapter 6 We Distribute, Yet Things Multiply

6.1 Some Properties of Multiplication

Figure It Out (Pages 142-143)

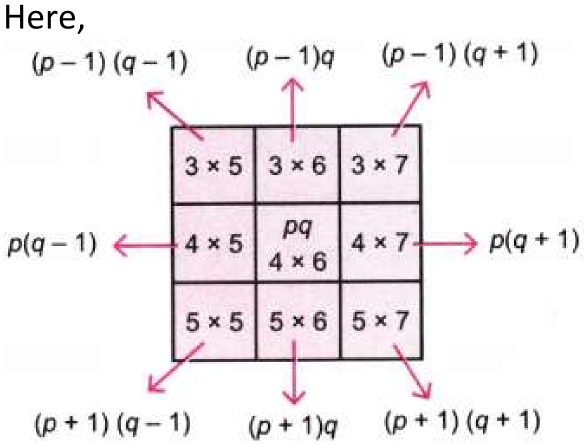
Question 1. Observe the multiplication grid below. Each number inside the grid is formed by multiplying two numbers. If the middle number of a 3 × 3 frame is given by the expression pq, as shown in the figure, write the expressions for the other numbers in the grid.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

3 × 5	3 × 6	3 × 7
4 × 5	4 × 6	4 × 7
5 × 5	5 × 6	5 × 7

	pq	

Solution:
Here,



Question 2. Expand the following products.
(i) (3 + u) (v – 3)
(ii) 23(15 + 6a)
(iii) (10a + b) (10c + d)
(iv) (3 – x)(x – 6)
(v) (-5a + b) (c + d)
(vi) (5 + z) (y + 9)

Solution: (i) We have, $(3 + u)(v - 3)$
$= 3(v - 3) + u(v - 3)$
$= 3v - 9 + uv - 3u$
$= 3v - 3u + uv - 9$

(ii) Here, $23(15 + 6a)$
$= 23 \times 15 + 23 \times 6a$
$= 10 + 4a$

(iii) Here, $(10a + b)(10c + d)$
$= 10a \times 10c + 10a \times d + b \times 10c + b \times d$
$= 100ac + 10ad + 10bc + bd$

(iv) Here, $(3 - x)(x - 6)$
$= 3(x - 6) - x(x - 6)$
$= 3x - 18 - x^2 + 6x$
$= -x^2 + 9x - 18$

(v) We have, $(-5a + b)(c + d)$
$= -5a(c + d) + b(c + d)$
$= -5ac - 5ad + bc + bd$

(vi) We have, $(5 + z)(y + 9)$
$= 5(y + 9) + z(y + 9)$
$= 5y + 45 + yz + 9z$

Question 3. Find 3 examples where the product of two numbers remains unchanged when one of them is increased by 2 and the other is decreased by 4.
Solution: Let the numbers be a and b.
Then, $ab = (a + 2)(b - 4)$
$\Rightarrow ab = ab - 4a + 2b - 8$
$\Rightarrow ab - ab + 4a + 8 = 2b$
$\Rightarrow 4a + 8 = 2b$ [Divide throughout by 2]
$\Rightarrow 2a + 4 = b$
$\Rightarrow b = 2a + 4$
For $a = 1$, $b = 2 \times 1 + 4 = 6$
$ab = 1 \times 6 = 6$
and $(a + 2)(b - 4) = 3 \times 2 = 6$
Hence, $ab = (a + 2)(b - 4)$
Let $a = 2$, then $b = 2 \times 2 + 4 = 8$
Let $a = 3$, then $6 = 2 \times 3 + 4 = 10$
Three such pairs are 1 and 6, 2 and 8, and 3 and 10.

Question 4. Expand (i) $(a + ab – 3b^2) (4 + b)$, and (ii) $(4y + 7) (y + 11z – 3)$.

Solution: (i) Here, $(a + ab – 3b^2) (4 + b)$

$= (4 + b) (a + ab – 3b^2)$

$= 4(a + ab – 3b^2) + b(a + ab – 3b^2)$

$= 4a + 4ab – 12b^2 + ab + ab^2 – 3b^3$

$= 4a + 5ab – 12b^2 + ab^2 – 3b^3$

(ii) Here, $(4y + 7) (y + 11z – 3)$

$= 4y(y + 11z – 3) + 7(y + 11z – 3)$

$= 4y2 + 44yz – 12y + 7y + 77z – 21$

$= 4y2 + 44yz – 5y + 77z – 21$

Question 5. Expand (i) $(a – b) (a + b)$, (ii) $(a – b) (a^2 + ab + b^2)$, and (iii) $(a – b) (a^3 + a^2b + ab^2 + b^3)$. Do you see a pattern? What would be the next identity in the pattern that you see? Can you check it by expanding?

Solution: (i) Here, $(a – b) (a + b)$

$= a(a + b) – b(a + b)$

$= a^2 + ab – ab – b^2$

$= a^2 – b^2$

(ii) Here, $(a – b) (a^2 + ab + b^2)$

$= a(a^2 + ab + b^2) – b(a^2 + ab + b^2)$

$= a^3 + a^2b + ab^2 – a^2b – ab^2 – b^3$

$= a^3 – b^3$

(iii) We have, $(a – b) (a^3 + a^2b + ab^2 + b^3)$

$= a(a^3 + a^2b + ab^2 + b^3) – b(a^3 + a^2b + ab^2 + b^3)$

$= a^4 + a^3b + a^2b^2 + ab^3 – a^3b – a^2b^2 – ab^3 – b^4$

$= a^4 – b^4$

We observe the following pattern $(a – b) (a^n + a^{n-1} b + ... + b^n) = a^{n+1} – b^{n+1}$

Next identity in the pattern would be $(a – b) (a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 – b^5$

6.2 Special Cases of the Distributive Property

Figure It Out (Page 149)

Question 1. Which is greater: $(a – b)^2$ or $(b – a)^2$? Justify your answer.

Solution: Here, $(a – b)^2 = a^2 + b^2 – 2ab$ ....(1)

and $(b – a)^2 = b^2 + a^2 – 2ba$

$b^2 + a^2 = a^2 + b^2$ and $ba = ab$

$(b – a)^2 = a^2 + b^2 – 2ab$ ....(2)

Comparing (1) and (2), we get $(a – b)^2 = (b – a)^2$

Question 2. Express 100 as the difference of two squares.

Solution: $a^2 - b^2 = 100$

$(a + b)(a - b) = 100$

$[100 = 1 \times 100, 2 \times 50, 4 \times 25, 5 \times 20, 10 \times 10]$

We can take anyone

Let us take $50 \times 2 = 100$

Hence, $(a + b)(a - b) = 50 \times 2$

$a + b = 50$ ....(1)

$a - b = 2$ ....(2)

Adding (1) and (2)

$2a = 52$

$\Rightarrow a = 26$

Substituting $a = 26$ in (1)

$26 + b = 50$

$\Rightarrow b = 50 - 26 = 24$

Let us check $26^2 - 24^2 = 676 - 576 = 100$

Hence $26^2 - 24^2 = 100$

Question 3. Find 406^2, 72^2, 145^2, 1097^2, and 124^2 using the identities you have learned so far.

Solution: (i) $406^2 = (400 + 6)^2$

$= 400^2 + 2 \times 400 \times 6 + 6^2$

$= 160000 + 4800 + 36$

$= 164836$

(ii) $72^2 = (50 + 22)^2$

$= 50^2 + 2 \times 50 \times 22 + 22^2$

$= 2500 + 2200 + 484$

$= 5184$

(iii) $145^2 = (150 - 5)^2$

$= 150^2 - 2 \times 150 \times 5 + 5^2$

$= 22500 - 1500 + 25$

$= 21025$

(iv) $1097^2 = (1100 - 3)^2$

$= 1100^2 - 2 \times 1100 \times 3 + 3^2$

$= 1210000 - 6600 + 9$

$= 1203409$

(v) $124^2 = (100 + 24)^2$

$= 100^2 + 2 \times 100 \times 24 + 24^2$

$= 10000 + 4800 + 576$

$= 15376$

Question 4. Do Patterns 1 and 2 hold only for counting numbers? Do they hold for negative integers as well? What about fractions? Justify your answer.
Solution: Pattern 1
$2(a^2 + b^2) = (a + b)^2 + (a – b)^2$

Case-I
Let a = 4, b = 2
LHS = $2(4^2 + 2^2) = 2 \times (16 + 4) = 40$
RHS = $(4 + 2)^2 + (4 – 2)^2 = 36 + 4 = 40$
∴ Pattern 1 holds for counting numbers.

Case-II
Let a = -4, b = -2
LHS = $2((-4)^2 + (-2)^2)$
$= 2 \times (16 + 4)$
$= 2 \times 20$
$= 40$
RHS = $(-4 + (-2))^2 + (-4 – (-2))^2$
$= (-4 – 2)^2 + (-4 + 2)^2$
$= (-6)^2 + (-2)^2$
$= 36 + 4$
$= 40$
LHS = RHS
∴ Pattern 1 holds for negative integers also.

Case-III
Let $a = \frac{1}{2}$, $b = \frac{1}{3}$

$$\text{LHS} = 2\left(\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2\right)$$

$$= 2\left[\frac{1}{4} + \frac{1}{9}\right] = 2 \times \frac{13}{36} = \frac{13}{18}$$

$$\text{RHS} = \left(\frac{1}{2} + \frac{1}{3}\right)^2 + \left(\frac{1}{2} - \frac{1}{3}\right)^2$$

$$= \left(\frac{5}{6}\right)^2 + \left(\frac{1}{6}\right)^2 = \frac{25}{36} + \frac{1}{36}$$

$$= \frac{26}{36} = \frac{13}{18}$$

The pattern holds for fractions also.

Pattern 2
$a^2 – b^2 = (a + b) (a – b)$
Case-I
Let a = 5, b = 3
LHS = $5^2 – 3^2 = 25 – 9 = 16$

RHS = (5 + 3) (5 – 3) = 8 × 2 = 16
∴ LHS = RHS
∴ Pattern 2 holds for counting numbers.

Case-II
Let a = -5, b = -3
Now, LHS = $(-5)^2 – (-3)^2$ = 25 – 9 = 16
and RHS = [(-5) + (-3)] [(-5) – (-3)]
= (-5 – 3) (-5 + 3)
= (-8) (-2)
= 16
∴ LHS = RHS
∴ Pattern 2 holds for negative integers also.

Case-III
Let a = 12, b = 13
LHS = (12)2−(13)2
= 14−19
= 9−436
= 536
and RHS = (12+13)(12−13)=(3+26)(3−26)=56×16=536
∴ LHS = RHS
∴ Pattern 2 holds for fractions also.
6.3 Mind the Mistake, Mend the Mistake, 6.4 This Way or That Way, All Ways Lead to the Bay

Figure It Out (Pages 154-156)

Question 1. Compute these products using the suggested identity.
(i) 46^2 using Identity 1A for $(a + b)^2$
(ii) 397 × 403 using Identity 1C for (a + b) (a – b)
(iii) 91^2 using Identity 1B for $(a – b)^2$
(iv) 43 × 45 using Identity 1C for (a + b) (a – b)

Solution: (i) $46^2 = (40 + 6)^2$
= $40^2 + 2 \times 40 \times 6 + 6^2$ [∵ $(a + b)^2 = a^2 + 2ab + b^2$]
= 1600 + 480 + 36
= 2116

(ii) 397 × 403 = (400 – 3) (400 + 3) [∵ $(a + b) \times (a – b) = a^2 – b^2$]
= $400^2 – 3^2$
= 160000 – 9
= 159991

(iii) $91^2 = (100 – 9)^2$
$= 100^2 – 2 \times 100 \times 9 + 9^2$ [$\because (a – b)^2 = a^2 + b^2 – 2ab$]
= 10000 – 1800 + 81
= 8281

(iv) $43 \times 45 = (44 – 1)(44 + 1)$ [$\because a^2 – b^2 = (a + b) \times (a – b)$]
$= 44^2 – 1^2$
= 1936 – 1
= 1935

Question 2. Use either a suitable identity or the distributive property to find each of the following products.
(i) $(p – 1)(p + 11)$
(ii) $(3a – 9b)(3a + 9b)$
(iii) $-(2y + 5)(3y + 4)$
(iv) $(6x + 5y)^2$
(v) $(2x – 12)^2$
(vi) $(7p) \times (3r) \times (p + 2)$

Solution: (i) $(p – 1)(p + 11)$
$= p(p + 11) – 1(p + 11)$
$= p^2 + 11p – p – 11$
$= p^2 + 10p – 11$

(ii) $(3a – 9b)(3a + 9b)$
$= (3a)^2 – (9b)^2$
$= 9a^2 – 81b^2$

(iii) $-(2y + 5)(3y + 4)$
$= (-2y – 5)(3y + 4)$
$= -2y(3y + 4) – 5(3y + 4)$
$= -6y^2 – 8y – 15y – 20$
$= -6y^2 – 23y – 20$

(iv) $(6x + 5y)^2$
$= (6x)^2 + 2(6x)(5y) + (5y)^2$
$= 36x^2 + 60xy + 25y^2$

(v) $(2x – 12)2$
$= (2x)^2 – 2 \times 2x \times 12 + (12)2$
$= 4x^2 – 2x + 14$

(vi) $(7p) \times (3r) \times (p + 2)$
$= 7p \times 3r \times (p + 2)$

$= 21pr(p + 2)$
$= 21pr \times p + 21pr \times 2$
$= 21p^2r + 42pr$

Question 3.
For each statement, identify the appropriate algebraic expression(s).
(i) Two more than a square number.

- $2 + s$
- $(s + 2)^2$
- $s^2 + 2$
- $s^2 + 4$
- $2s^2$
- 2^2s

(ii) The sum of the squares of two consecutive numbers

- $m^2 + n^2$
- $(m + n)^2$
- $m^2 + 1$
- $m^2 + (m + 1)^2$
- $m^2 + (m - 1)^2$
- $(m + (m + 1))^2$
- $(2m)^2 + (2m + 1)^2$

Solution: (i) Two more than a square is $s^2 + 2$
(ii) Sum of the squares of two consecutive numbers is $m^2 + (m + 1)^2$

Question 4. Consider any 2 by 2 square of numbers in a calendar, as shown in the figure.

February						
Su	M	Tu	W	Th	F	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

Find products of numbers lying along each diagonal – 4 × 12 = 48, 5 × 11 = 55. Do this for the other 2 by 2 squares. What do you observe about the diagonal products? Explain why this happens.
Hint: Label the numbers in each 2 by 2 square as

a	$(a + 1)$
$a + 7$	$(a + 8)$

Solution:
Case-I

6	7
13	14

Here, 6 × 14 = 84
13 × 7 = 91
Difference = 91 – 84 = 7

Case-II

9	10
16	17

Here, 9 × 17 = 153
16 × 10 = 160
Difference = 160 – 153 = 7
We observe that the difference of the diagonal products in both cases is always 7.

Question 5. Verify which of the following statements are true.
(i) (k + 1) (k + 2) – (k + 3) is always a multiple of 2.
(ii) (2q + 1) (2q – 3) is a multiple of 4.
(iii) Squares of even numbers are multiples of 4, and squares of odd numbers are 1 more than multiples of 8.
(iv) $(6n + 2)^2 – (4n + 3)^2$ is 5 less than a square number.

Solution: (i) (k + 1)(k + 2) – (k + 3) is always a multiple of 2
Let k = 5, Then (5 + 1) (5 + 2) – (5 + 3)
= 6 × 7 – 8
= 42 – 8
= 34
34 is a multiple of 2.
∴ The statement is true.

(ii) (2q + 1) (2q – 3) is a multiple of 4.
Let q = 3, Then (6 + 1) (6 – 3)
= 7 × 3
= 21
21 is not a multiple of 4
∴ The statement is false.

(iii) The square of an even number is a multiple of 4.
$2^2 = 4 = 4 \times 1$
$4^2 = 16 = 4 \times 4$

$6^2 = 36 = 4 \times 9$
∴ The statement is true.
The square of an odd number is 1 more than a multiple of 8.
$3^2 = 9 = 8 \times 1 + 1$
$5^2 = 25 = 8 \times 3 + 1$
$7^2 = 49 = 8 \times 6 + 1$
∴ The statement is true.

(iv) $(6n + 2)^2 – (4n + 3)^2$ is 5 less than a square number.
Let n = 2, $(6 \times 2 + 2)^2 – (4 \times 2 + 3)^2$
$= 14^2 – 11^2$
= 196 – 121
= 75
= 80 – 5
But 80 is not a square number.
∴ The statement is false.

Question 6. A number leaves a remainder of 3 when divided by 7, and another number leaves a remainder of 5 when divided by 7. What is the remainder when their sum, difference, and product are divided by 7?
Solution: Let the numbers be x and y.
x = 7a + 3, y = 7b + 5

Sum = x + y
= 7a + 3 + 7b + 5
= 7(a + b) + 8
= 7(a + b) + 7 + 1
= 7(a + b + 1) + 1
∴ The remainder on division by 7 is 1.

Difference = x – y
= (7a + 3) – (7b + 5)
= 7a + 3 – 7b – 5
= 7(a – b) – 2
= 7(a – b) – 1 + 5 (∵ -2 = -7 + 5)
= 7(a – b – 1) + 5
∴ The remainder on division by 7 is 5.

Product = xy
= (7a + 3) (7b + 5)
= 49ab + 35a + 21b + 15
= (49ab + 35a + 21b + 14) + 1
= 7(7ab + 5a + 3b + 2) + 1
∴ The remainder on division by 7 is 1.

Question 7. Choose three consecutive numbers, square the middle one, and subtract the product of the other two. Repeat the same with other sets of numbers. What pattern do you notice? How do we write this as an algebraic equation? Expand both sides of the equation to check that it is a true identity.

Solution: Let us take the numbers 7, 8, 9

Now, $8^2 - 7 \times 9 = 64 - 63 = 1$

Let us take the numbers 10, 11, 12

Then $11^2 - 10 \times 12 = 121 - 120 = 1$

Generalizing:

Let the numbers be a – 1, a, a + 1

Then $a^2 - (a + 1)(a - 1) = 1$

LHS = $a^2 - (a + 1)(a - 1)$

= $a^2 - (a^2 - 1)$

= $a^2 - a^2 + 1$

= 1

∴ LHS = RHS

Hence, the identity is correct.

Question 8. What is the algebraic expression describing the following steps: add any two numbers? Multiply this by half of the sum of the two numbers? Prove that this result will be half of the square of the sum of the two numbers.

Solution: Let the two numbers be a and b.

Step 1: a + b

Step 2: (a + b) × 12 (a + b)

∴ $(a + b) \times 12 (a + b) = 12 (a + b)^2$

Question 9. Which is larger? Find out without fully computing the product.

(i) 14 × 26 or 16 × 24

(ii) 25 × 75 or 26 × 74

Solution: (i) Let p = 14 × 26

p’ = 16 × 24

= (14 + 2) (26 – 2)

= 14 × 26 + 2 × 26 – 14 × 2 – 2 × 2

= 14 × 26 + 2(26 – 14 – 2)

= 14 × 26 + 2 × 10

p’ = p + 2 × 10

∴ p’ > p or 16 × 24 > 14 × 26

(ii) Let p = 25 × 75

p’ = 26 × 74

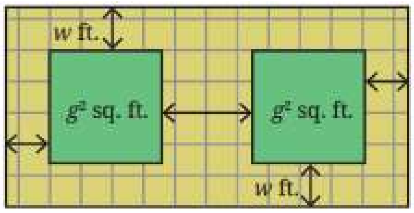
= (25 + 1) (75 – 1)

= 25 × 75 + 75 × 1 – 25 × 1 – 1 × 1

= p + (75 – 25 – 1)

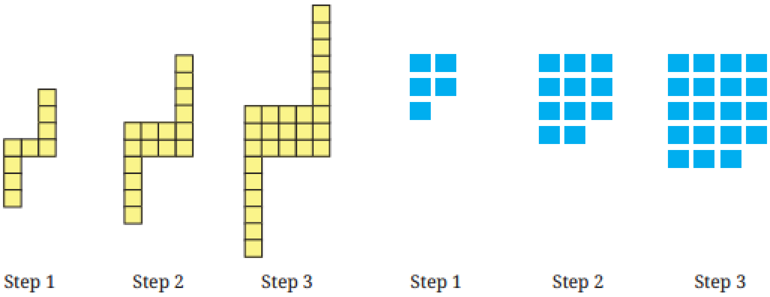
$= p + 49$
$\therefore p' > p$ or $26 \times 74 > 25 \times 75$

Question 10. A tiny park is coming up in Dhauli. The plan is shown in the figure. The two square plots, each of area g2 sq. ft., will have a green cover. All the remaining area is a walking path w ft. wide that needs to he tiled. Write an expression for the area that needs to be tiled.

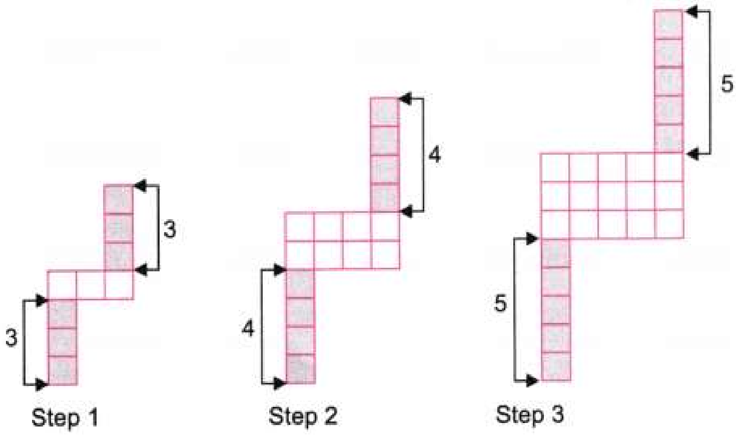


Solution: Length = w + g + 2w + g + w = 4w + 2g
Breadth = w + g + w = 2w + g
Area of park = (4w + 2g) (2w + g)
$= 8w^2 + 4wg + 4wg + 2g^2$
$= 8w^2 + 8wg + 2g^2$
Area of path = Area of park – Area of green cover
$= 8w^2 + 8wg + 2g^2 – 2g^2$
$= 8w^2 + 8wg$
$\therefore (8w^2 + 8wg)$ sq. feet area needs to be tiled.

Question 11. For each pattern shown below,
(i) Draw the next figure in the sequence.
(ii) How many basic units are there in Step 10?
(iii) Write an expression to describe the number of basic units in Step y.

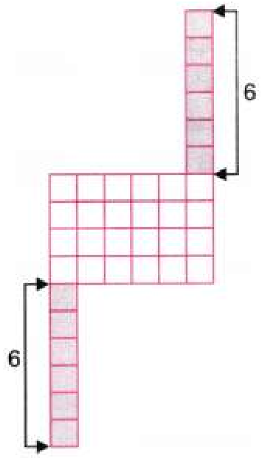


Solution: (a)



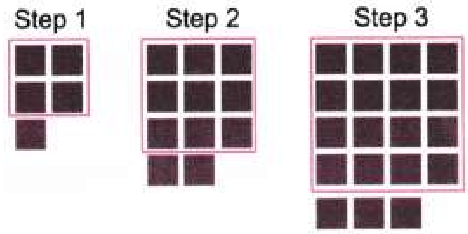
Step 1: 2 vertical strips of 3 units each + 1 vertical strip of 3 units
= 3 strips of 3 units each

= 9 units squares
= $(1 + 2)^2$ unit squares
Step 2: 4 strips of 4 units each
= 16 units squares
= $(2 + 2)^2$ unit squares
Step 3: 5 strips of 5 units each
= 25 units squares
= $(3 + 2)^2$ unit squares

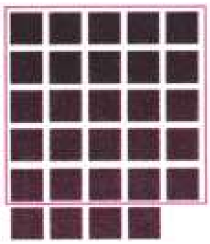


Step 4: (i) 6 strips of 6 units each = 2 are vertical and 4 are horizontal
(ii) Number of unit squares in step 10 = $(10 + 2)^2$ = 144
(iii) Number of unit squares in step y = $(y + 2)^2$

(b) (i)



Number of unit squares in step 1 = 5 = $2^2 + 1$
Number of unit squares in step 2 = 11 = $3^2 + 2$
Number of unit squares in step 3 = 19 = $4^2 + 3$



(ii) Step 1 has $(1 + 1)^2 + 1$ or 5 squares
Step 2 has $(2 + 1)^2 + 2$ or 11 squares
Step 3 has $(3 + 1)^2 + 3$ or 19 squares
Hence step 10 has $(10 + 1)^2 + 10$ or 131 squares
(iii) Step y has $[(y + 1)^2 + y]$ squares